RESONANT REFLECTIONS IN A PARTICLE CONDUCTING THROUGH A NORMAL-CONDUCTOR–SUPERCONDUCTOR INTERFACE IN MAGNETIC FIELDS

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A quantum resonance in the conduction through the normal-conductor–superconductor junction is investigated in the presence of a magnetic field using a numerical transfer matrix method. It is shown that dip-pairs in the conductance curve occur at a certain geometry of the system owing to the resonant reflection caused by the interference of reflected electron and hole waves under a finite magnetic field. By examining a local density of quasiparticles, we found that the feature of the resonant state giving the dip-pair in the conductance in a magnetic field is very different from that in a finite bias voltage.

1. Introduction

The advanced fabrication technology has made it possible to create various nanostuctures including a good quality normal-conductor–superconductor (NS) interface.1–7 Many novel phenomena have been reported on these composite systems.1–4 For example, the observation of a quantization of critical current was reported on a superconducting quantum point contact consisting of a split-gate superconductor-2DEG-superconductor junction.2 Furthermore, a resonant structure resulting from the quantum interference of quasiparticles at the step edge was presented in that study.2 Several numerical researches using the modal matching method and the recursive Green’s function method have been performed to investigate these phenomena.8,9

The conduction through the NS junction is known to be strongly influenced by the Andreev reflection10 of the quasiparticles at the NS interface,11 which brings to the novel phenomena such as those mentioned above. The quasiparticle reflected by the Andreev reflection has a retroproperty, i.e., that exactly follows the trajectory of the incident quasiparticle. In the real system, the Andreev reflection is not perfect, which induces a certain amount of the normal reflection. These reflections bring about quantum effects such as the quantum resonance. When a bias voltage is applied to the NS interface, the retroproperty is disrupted. By numerical investiga-
tion, the resonant conduction was predicted, which was caused by the interference of an electron and a hole waves under a finite bias voltage.\textsuperscript{8,12} The retroproperty of the quasiparticles is also disrupted in a finite magnetic field, because the direction of the force acting on an electron-like quasiparticle is opposite to that on a hole-like quasiparticle in a magnetic field.

It was reported that the transmission resonance in the NS system with a cavity in a finite voltage brought about the perfect reflection which gave rise to the dip-pair in the conductance curve.\textsuperscript{8,12} In the previous study, we investigated the quantum resonant state causing the dip pair in the conductance in the NS system with the narrow-wide (NW) geometry under a finite bias voltage by the modal matching method.\textsuperscript{12} In that study, we showed the local densities of the electron-like and hole-like quasiparticles when the resonance occurs.\textsuperscript{12} The occurrence of the resonant reflection is also expected in such a structure in a magnetic field. However, to our knowledge, detailed investigation of the resonant state in a magnetic field in such a system has not been performed yet. Thus, in this study, we examine the effect of a magnetic field on the transmission resonance, by calculating the distribution of the quasiparticles in the resonant state. To calculate the conductance and the electronic state in a magnetic field, we use the transfer matrix method used with the \textit{stabilized iterative technique}, which is an extension of the method to calculate the conductance of the normal quantum wire.\textsuperscript{13} We treat the quasi one-dimensional wire with a NW geometry including the NS interface. It is known that the quantum resonant state is sensitive to the change of sample geometry,\textsuperscript{8,12} and thus, we investigate how the quantum interference effect of the quasiparticles experiencing the Andreev and normal reflections depends on the sample geometry. To clarify the geometrical effects of the wire on the conductance further, we also consider a simple case, where either the narrow or the superconducting region is deleted.

This paper is organized as follows: The details of the model and the method used in our calculations are presented in Sec. 2. The numerical results for the calculation are given in Sec. 3. In Sec. 3.1, we show the conductance and the resonant state in a bias voltage, and then the effect of a magnetic field on transport properties is shown in Sec. 3.2. The results of the examination of the effect of disorder on the transmission resonance are given in Sec. 3.3. We also treat the simple case to clarify the geometrical effect, and the results are shown in Secs. 3.4 and 3.5. A summary is given in Sec. 4.

2. Model and Method

We consider phase-coherent transport phenomena of the quasiparticles in the NS system, which is described by the Bogoliubov-de Gennes (BdG) equation

\[ \mathcal{H} \left( \begin{array}{c} u \left( r \right) \\ v \left( r \right) \end{array} \right) = \left[ \begin{array}{cc} \mathcal{H}_0 & \Delta \left( r \right) \\ \Delta^* \left( r \right) & -\mathcal{H}_0 \end{array} \right] \left( \begin{array}{c} u \left( r \right) \\ v \left( r \right) \end{array} \right) = \varepsilon \left( \begin{array}{c} u \left( r \right) \\ v \left( r \right) \end{array} \right) , \right. \]

where \( u(r) \) and \( v(r) \) are the wave functions of electron-like and hole-like quasiparticles. \( \Delta \) is the pair potential amplitude. The single-particle Hamiltonian is

\[ \mathcal{H}_0 = \left[ p + eA \right]^2 / 2m + U(x,y) - \mu, \]
Fig. 1. Schematic illustration of the system. The quantities $N$ and $M$ are the numbers of the lattice site for the $x$ and $y$ directions. The potential barriers and constriction exist in the region $1 \leq j \leq N$. The shaded region is the superconducting region, if $\Delta \neq 0$.

where $A$ and $U$ are the vector and scalar potentials, respectively.

To examine the interference of particles scattered by atomic impurities and the NS interface, we consider a quantum wire described by a two-dimensional tight-binding model as illustrated in Fig. 1, where the lattice sites are labeled with the coordinate $(j, m)$. The wire is assumed to be infinitely long in the $x$-direction, but in the $y$-direction it is assumed to be finite and consists of $M$ lattice sites. The single-particle Hamiltonian in this lattice model can be described as

$$
\mathcal{H}_0 = -t \sum_j \sum_{m=1}^M \left( c_{j+1,m}^\dagger c_{j,m} p_m + \text{H.c.} \right)
$$

$$
- t \sum_j \sum_{m=1}^{M-1} \left( c_{j,m+1}^\dagger c_{j,m} + \text{H.c.} \right)
$$

$$
+ \sum_j \sum_{m=1}^M \mu_j c_{j,m}^\dagger c_{j,m} + \sum_{j=1}^N \sum_{m=1}^M v_{j,m}^p c_{j,m}^\dagger c_{j,m},
$$

(2)

where $c_{j,m}^\dagger$ is the creation operator for an electron, $v_{j,m}^p$ is the scattering potential at the site $(j, m)$, and $t (> 0)$ is the hopping integral between the nearest neighbor sites. The quantity $\mu_j$ is given by

$$
\mu_j = \begin{cases} 
4t - \mu_N & \text{for } j \leq N \\
4t - \mu_S & \text{for } j > N 
\end{cases}
$$

(3)

When a uniform magnetic field $-B$ is applied along the $z$-direction, the Peierls phase factor can be chosen as

$$
p_m = \exp \left[ 2\pi i \tilde{B} \{ m - (M + 1)/2 \} \right],
$$

where $\tilde{B} = Ba^2/\phi_0$ with $a$ being the lattice constant and $\phi_0 (= h/e)$ is the magnetic flux quantum. We take $\phi_0/a^2$ as a unit of magnetic field. The BdG equation in this
lattice model is rewritten as,

$$\langle j, m | \mathcal{H} | \psi \rangle = \varepsilon \langle j, m | \psi \rangle,$$

where

$$|\psi\rangle = \sum_{j,m} a_{j,m} |j, m\rangle,$$

$$|j, m\rangle = c_{j,m}^\dagger |0\rangle,$$

and

$$a_{j,m} = \begin{pmatrix} u_{j,m} \\ v_{j,m} \end{pmatrix},$$

with $|0\rangle$ being vacuum. Using Eq. (4), we can write the equation of motion as

$$(\varepsilon 1 - H_j) a_j + P_j a_{j-1} + P_j^* a_{j+1} = 0,$$

where

$$a_j = \begin{pmatrix} u_j \\ v_j \end{pmatrix}.$$

The column vectors $u_j$ and $v_j$ are the wave functions of electron-like and hole-like quasiparticles in the $j-$th slice, respectively. The matrix $H_j$ is given by

$$H_j = \begin{pmatrix} H_j^a & H_j^b \\ H_j^b & -H_j^a \end{pmatrix},$$

where

$$H_j^a = \begin{pmatrix} \mu_j + v_{j,1}^p & -t & 0 & \ldots & 0 \\
-t & \mu_j + v_{j,2}^p & -t & \ldots & 0 \\
0 & -t & \mu_j + v_{j,3}^p & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & \mu_j + v_{j,M}^p \end{pmatrix},$$

$$H_j^b = \begin{pmatrix} \Delta \delta_{l,m} & \text{for } j > N \\
0 & \text{otherwise} \end{pmatrix}.$$

The $2M \times 2M$ diagonal matrix $P_j$ is given by

$$(P_j)_{l,m} = \begin{cases} p_m & (l = m = 1, \ldots, M \text{ and } j \leq N) \\
-p_m & (l = m = M + 1, \ldots, 2M \text{ and } j \leq N) \\
0 & \text{otherwise} \end{cases}.\,$$

The amplitude of the site $j = N + 1$ is related to that of the site $j = 0$ as

$$\begin{pmatrix} a_N \\ a_{N+1} \end{pmatrix} = T_N T_{N-1} \ldots T_1 \begin{pmatrix} a_0 \\ a_1 \end{pmatrix},$$

where

$$T_j = \begin{pmatrix} 0 \\ -P_j^2 \end{pmatrix} t^{-1} P_j (H_j - \varepsilon 1).$$

Using equation $a_j = \lambda a_{j-1}$ with $\lambda$ being equal to $e^{ika}$, we can obtain the column vector as linear independent solutions of the following eigenvalue problem:

$$\begin{pmatrix} 0 \\ -P_j^2 \\ t^{-1} P_j (H_j - \varepsilon 1) \end{pmatrix} \begin{pmatrix} a_{j-1} \\ a_j \end{pmatrix} = \lambda \begin{pmatrix} a_{j-1} \\ a_j \end{pmatrix}.$$
This equation has \(4M\) eigenvalues and \(4M\) eigenvectors, which are classified into \(M\) right-going and left-going electron as well as hole waves, respectively. Employing eigenvectors at the \(j\)-th slice \((u_j^\pm)\) and \((v_j^\pm)\) obtained, we define

\[
U_N^\pm = \begin{pmatrix} u_0^\pm & 0 \\ 0 & v_0^\pm \end{pmatrix} \quad \text{and} \quad U_S^\pm = \begin{pmatrix} u_{N+1}^\pm & (u_{N+1}^\pm)^* \\ (v_{N+1}^\pm)^* & v_{N+1}^\pm \end{pmatrix},
\]

where

\[
u_j^\pm = \begin{bmatrix} (u_j^\pm)_1, \ldots, (u_j^\pm)_M \end{bmatrix}, \quad \nu_j^\pm = \begin{bmatrix} (v_j^\pm)_1, \ldots, (v_j^\pm)_M \end{bmatrix}.
\]

Furthermore, we define the \(2M \times 2M\) matrices \(\Lambda_N^\pm\) and \(\Lambda_S^\pm\) using the eigenvalues \((\lambda_j^\pm)\) and \((\lambda_{N+1}^\pm)\)

\[
\Lambda_N^\pm = \begin{pmatrix} \lambda_0^{\epsilon \pm} & 0 \\ 0 & \lambda_0^{h \pm} \end{pmatrix} \quad \text{and} \quad \Lambda_S^\pm = \begin{pmatrix} \lambda_{N+1}^{\epsilon \pm} & 0 \\ 0 & \lambda_{N+1}^{h \pm} \end{pmatrix},
\]

where

\[
\begin{bmatrix} \lambda_j^{\epsilon (h) \pm} \end{bmatrix} = \text{diag}\left[(\lambda_j^{\epsilon (h) \pm})_1, \ldots, (\lambda_j^{\epsilon (h) \pm})_M\right].
\]

A left-going and right-going waves are written at \(j = 0, 1\) and \(j = N, N + 1\) as

\[
\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} U_N^+ & U_N^- \\ U_N^+ \Lambda_N^\pm & U_N^- \Lambda_N^\pm \end{pmatrix} \begin{pmatrix} L^+ \\ L^- \end{pmatrix} = T_0 \begin{pmatrix} L^+ \\ L^- \end{pmatrix},
\]

\[
\begin{pmatrix} a_N \\ a_{N+1} \end{pmatrix} = \begin{pmatrix} U_S^+ & U_S^- \\ U_S^+ \Lambda_S^\pm & U_S^- \Lambda_S^\pm \end{pmatrix} \begin{pmatrix} R^+ \\ R^- \end{pmatrix} = T_S \begin{pmatrix} R^+ \\ R^- \end{pmatrix},
\]

where

\[
L^\pm = \begin{pmatrix} L^\epsilon_{\pm} \\ L^h_{\pm} \end{pmatrix} \quad \text{and} \quad R^\pm = \begin{pmatrix} R^\epsilon_{\pm} \\ R^h_{\pm} \end{pmatrix}
\]

are the appropriate vectors consisting of expansion coefficients. The channel coefficients \(R\) and \(L\) are represented by the transfer matrix relations or the scattering matrix as

\[
\begin{pmatrix} R^+ \\ R^- \end{pmatrix} = T_S^{-1} T_N T_{N-1} \ldots T_1 T_0 \begin{pmatrix} L^+ \\ L^- \end{pmatrix},
\]

\[
\begin{pmatrix} R^+ \\ L^- \end{pmatrix} = \begin{pmatrix} t & r' \\ r & t' \end{pmatrix} \begin{pmatrix} L^+ \\ R^- \end{pmatrix}.
\]

Here the \(2M \times 2M\) matrices of the transmission \(t\) and reflection \(r\) coefficients are expressed as

\[
t = \begin{pmatrix} t_{\epsilon \epsilon} & t_{\epsilon h} \\ t_{h \epsilon} & t_{h h} \end{pmatrix} \quad \text{and} \quad r = \begin{pmatrix} r_{\epsilon \epsilon} & r_{\epsilon h} \\ r_{h \epsilon} & r_{h h} \end{pmatrix},
\]

where \(r_{\epsilon \epsilon}\) (\(r_{\epsilon h}\)) denotes the matrix of the reflection amplitude for a particle incident as an electron from the lead of normal-conductor, \(i.e., j \leq 0\), and reflected as an electron (hole).
Next, we are concerned with the iteration method. To avoid the unstability of the transfer matrix calculation, we employ the following matrix iteration equation
\[ \begin{pmatrix} C_{1}^{j+1} & C_{2}^{j+1} \\ 0 & 1 \end{pmatrix} = T_{j} \begin{pmatrix} C_{1}^{j} & C_{2}^{j} \\ 0 & 1 \end{pmatrix} Q_{j} \quad \text{for} \quad 0 \leq j \leq N + 1, \] (27)
with
\[ T_{N+1} = \begin{pmatrix} 0 & (U_{S}^{+} A_{S}^{+})^{-1} \\ 1 & -U_{S}^{+} (U_{S}^{+} A_{S}^{+})^{-1} \end{pmatrix} \] (28)
and
\[ Q_{j} = \begin{pmatrix} 1 & 0 \\ Q_{j1} & Q_{j2} \end{pmatrix}. \] (29)

The $2M \times 2M$ matrices $Q_{j1}$ and $Q_{j2}$ are given by
\[ Q_{j1} = -Q_{j2} T_{j21} C_{1}^{j}, \] (30)
\[ Q_{j2} = \left( T_{j21} C_{2}^{j} + T_{j22} \right)^{-1}, \] (31)
where
\[ T_{j} = \begin{pmatrix} T_{j11} & T_{j12} \\ T_{j21} & T_{j22} \end{pmatrix}. \] (32)

We take as initial conditions of the iteration $C_{1}^{0} = 1$ and $C_{2}^{0} = 0$. We perform the iterative calculations from $j = 0$ to $j = N + 1$, which give the matrix of the transmission coefficients $t = C_{1}^{N+2}$. Every iteration procedure of this method includes the inversion of the matrix, which results in the stability of the calculations. Moreover, the matrix of the reflection coefficients $r$ is calculated by the matrix iteration equation
\[ \begin{pmatrix} D_{1}^{j+1} \\ D_{2}^{j+1} \end{pmatrix} = \begin{pmatrix} D_{1}^{j} \\ D_{2}^{j} \end{pmatrix} Q_{j}, \] (33)
\[ r = D_{1}^{N+2}. \] (34)

In this case, the initial conditions are $D_{1}^{0} = 0$ and $D_{2}^{0} = 1$. Finally, using the matrix $r$, we can calculate the conductance $G_{NS}$ by the following formula:
\[ G_{NS} = \frac{4e^{2}}{h} \left( N_{i} - \sum_{l,m} |(r_{ee})_{l,m}|^2 + \sum_{l,m} |(r_{he})_{l,m}|^2 \right), \] (35)

where $N_{i}$ is the total number of incident modes.

3. Conductance and Electronic State

3.1. Narrow-Wide/Super (NWS) system

Now, we move to the conductance calculated with eq. (35). We treat the narrow-wide geometry, in consideration of the experimental situation where particles are incident from the constriction. To this end, the narrow region, $1 \leq j \leq N_{\text{narrow}}$, is taken to be sufficiently long, i.e., $N_{\text{narrow}} a = 120 a$ and the width of the narrow
region is $M_n a = 40a$. The width of the other part of the wire is $Ma = 90a$. We employ a potential profile:

$$v_j = \begin{cases} 
12\mu_N & \text{for sites } 1 \leq j \leq 120 \text{ and } m \leq \frac{M-M_n}{2}, m > \frac{M+M_n}{2} \\
0 & \text{otherwise}
\end{cases}$$  \hspace{1cm} (36)

We have calculated the conductance as a function of $L/M_n$ with $L$ being $N-N_{\text{narrow}}$. The chemical potential in the normal region is $\mu_N = 0.018$ throughout this work. The parameters $\mu_S$ and $\Delta$ are $\mu_S = 6\mu_N$ and $\Delta = 0.1\mu_N$ in this case. Figure 2 (a) shows the typical examples of the conductance calculated in the cases $eV = 0$ and $eV/\mu_N = 0.04$ with $V$ being the bias voltage applied to the NS interface. As can be seen from Fig. 2 (a), at $eV = 0$, there are several peaks of the resonant tunneling because of the presence of the normal reflection at $j = N_{\text{narrow}}$ and the Andreev and normal reflections at $j = N + 1$. In Fig. 2 (a), the conductance curves show complicated structures even in the case at $eV = 0$, which is due to the occurrence of the multiple scatterings in the central region,

$$N_{\text{narrow}} < j \leq N.$$  

Although the number of propagating modes (subbands) in the narrow region, $0 < j \leq N_{\text{narrow}}$, is unity, the presence of three propagating modes and many evanescent modes for $j > N_{\text{narrow}}$ brings about the complex interference of the quasiparticles. The resonant tunneling can occur, in principle, for the first and the third modes in $N_{\text{narrow}} < j \leq N$. The width of the narrow region $M_n a$ is close to the interval between the neighboring nodes of the wave function of the third mode of the wide region. On the basis of such a size effect, the wave function of the third mode is considered to well-match that of the incident mode. Accordingly, the peaks occurring in the conductance are mainly due to the resonances of the third mode.

The resonance seems to occur when $L/M_n$ is about 0.9, 1.8, 2.8, and 3.7, since the interval between the neighboring resonances corresponds to half the Fermi-wavelength $\lambda_F$ of the third mode $0.93M_n$ at $eV = 0$.

Under a finite bias voltage at $eV/\mu_N = 0.04$, several peaks in the conductance curve change to dip-pairs [see Fig. 2 (a)]. The dip structure (the single dip or the dip-pair) in the conductance has also been reported in refs. 8, 16–20 for several geometries including the normal-conductor $t$–stab structure. The occurrence of dip is known to be related to the generation of the zero-pole pair of the Green function of the system.\(^8\),\(^18\),\(^19\) In this case, the wavelengths of electron-like and hole-like quasiparticles in the resonant states are different, so that the single dip splits into two.

Here, we discuss the parameter dependence of the conductance. Figures 2 (b) and 2 (c) show the conductances for the cases (b) $eV/\mu_N = 0.04$ and $\Delta/\mu_N = 0.05$, and (c) $eV/\mu_N = 0.06$ and $\Delta/\mu_N = 0.1$. As $eV/\Delta$ increases, the magnitude of splitting of the dip increases, because of an increase in the difference of the wavelengths of both quasiparticles in the resonant states. Moreover, the decrease of $\mu_S/\mu_N$ brings about a decrease in the amplitude of oscillation, since the probability of the normal reflection at NS interface reduces (not shown). These features are similar to those reported in Ref 8, although the geometry of the sample is different.
Fig. 2. The conductances are plotted as a function of $L/M_n$ (with $L = N - N_{\text{narrow}}$). Bias voltages are applied to the NS interface. (a) The dotted and solid lines show the conductances for $eV = 0$ and $eV/\mu_N = 0.04$, respectively. $\Delta/\mu_N$ equals to 0.1. Inset draws the schematic figure of the system in this case (see text). The closed region shows the superconducting region. (b) The dotted and solid lines show the conductances for $eV/\mu_N = 0.04$ and $eV/\mu_N = 0.06$, respectively. $\Delta/\mu_N$ equals to 0.1. (c) The dotted and solid lines show the conductances for $\Delta/\mu_N = 0.1$ and $\Delta/\mu_N = 0.05$, respectively. $eV/\mu_N$ equals to 0.04.
Fig. 3. The contour plot of modulus squared of the wave function of the quasiparticles for the resonant states (for the resonant reflection) around the central region in the case when the dip occurs in the conductance in Fig. 2 (a). The values \( L/M_n \) are set to \( L/M_n = 2.65 \) for (a) and (b), and \( L/M_n = 2.93 \) for (c) and (d). In (a) and (c), \( |u_{j,m}|^2 \) is plotted and in (b) and (d), \( |v_{j,m}|^2 \) is plotted. The interval of the contour lines is 0.1.
When the dip-pair occurs in the conductance curve shown in Fig. 2 (a), the probability amplitude of an electron or a hole is enhanced by the resonance. Figures 3 (a)-(d) depict the modulus squared of the single particle wave functions of electron and hole for the resonant states. In Figs. 3 (a) and 3 (b) [3 (c) and 3 (d)], the resonance of an electron-like (hole-like) quasiparticle leads to the enhancement of the probability amplitude of an electron (a hole).

### 3.2. Effect of Magnetic Field

Next, we discuss the effect of magnetic fields on the conductance. Figures 4 (a)-4 (d) show the $L$ dependence of the conductance for $B = 1 \times 10^{-5}, B = 2 \times 10^{-5}, B = 3 \times 10^{-5},$ and $B = 4 \times 10^{-5}$ (which correspond to $\hbar\omega_c/\mu_N = 0.007, \hbar\omega_c/\mu_N = 0.014, \hbar\omega_c/\mu_N = 0.021,$ and $\hbar\omega_c/\mu_N = 0.028$, respectively, with $\omega_c$ being the cyclotron frequency). The parameters $\mu_S$ and $\Delta$ are $\mu_S/\mu_N = 6$ and $\Delta/\mu_N = 0.1$, which are same as those used before. In finite magnetic fields at $eV = 0$, when the magnitude of $L$ is small ($L/M_n \leq 1.5$), the conductance curve is almost identical to that in the case at $B = 0$ [see Figs 4 (a)-(d)]. As $B$ increases, dips grow at about $L/M_n = 2.7$ and $L/M_n = 3.3$. When $B = 4 \times 10^{-5}$, these dips become dip-pairs, as shown in Fig. 4 (d). The location of the center of the dip-pair moves to left from the position of the conductance peak in the case at $B = 0$.

We have also examined the parameter dependence of the conductance. Figure 4(e) shows the conductance for the case $\mu_S/\mu_N = 3$ at $B = 4 \times 10^{-5}$. In this small $\mu_S/\mu_N$ case, the amplitude of the normal reflection on the NS interface is small, so that the magnitude of the oscillation is small [see Fig. 4(e)]. In Fig. 4 (e), the dip-pair can not be seen in the conductance curve, similarly the cases where $B \leq 3 \times 10^{-5}$ [see Fig. 4(a)-(c)]. It should be noted that the curve structure hardly changes when $\Delta$ varies in the superconducting region (not shown).

The dip-pair as shown in Fig. 4 (d) has also been seen in the case of the finite bias voltage at $B = 0$ [see Fig. 2 (a)]. However, the feature of the resonance when the dip occurs in a finite magnetic field case is quite different from that in the case of finite bias voltages at $B = 0$ [see Figs. 2 (a) and 3]. Figures 5 (a)-(d) show the modulus squared of the wave function when the conductance dips occur at $L/M_n = 2.58$ [see Figs. 5 (a) and 5 (b)] and $L/M_n = 2.7$ [see Figs. 5 (c) and 5 (d)]. As shown in Figs. 5 (a) and 5 (b), the calculated wave functions show that the total amount of the probability amplitude of electron [see Fig. 5 (a)] in the central region is similar to that of hole [see Fig. 5 (b)], indicating that the resonances of an electron-like and a hole-like quasiparticles take place simultaneously. Also, in Figs. 5 (c) and (d), the difference of the amounts of both quasiparticles is not so large, which also shows the resonance of both quasiparticles, though the amount of electron in the central region is somewhat larger than that of hole. The locations of the maxima of the probability amplitude in Figs. 5 (a) and (b) is similar to those in Figs. 5 (c) and (d). These features are very different from those of the resonance in a finite bias at $B = 0$. Moreover, as can be seen from Figs. 5 (a) and (b), when a dip occurs in the conductance, the mode (subband) of electron where the resonance occurs seems to be different from that of hole. At $L/M_n = 2.58$, for an electron, the third resonance of the third mode seems to occur and the forth resonance of
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Fig. 4. The conductances are plotted as a function of $L/M_n$ (with $L = N - N_{\text{narrow}}$). Magnetic fields are applied along the $z$-axis. The amplitudes of the magnetic field are (a) $B = 1 \times 10^{-5}$, (b) $B = 2 \times 10^{-5}$, (c) $B = 3 \times 10^{-5}$, (d) $B = 4 \times 10^{-5}$, and (e) $B = 4 \times 10^{-5}$ (which correspond to (a) $\hbar \omega_c/\mu N = 0.007$, (b) $\hbar \omega_c/\mu N = 0.014$, (c) $\hbar \omega_c/\mu N = 0.021$, (d) $\hbar \omega_c/\mu N = 0.028$, and (e) $\hbar \omega_c/\mu N = 0.028$, respectively). The values $\mu S/\mu N$ are (a)-(d) $\mu S/\mu N = 6$ and (e) $\mu S/\mu N = 3$. The second mode takes place for a hole. Due to the breakdown of the time reversal symmetry, the transmission of the first mode to the second mode can be permitted, and thus, the resonance of the second mode in the wide region can take place. In this case, the number of propagating modes in the narrow region is only one. The mixing of two electronic states with different parities can give rise to two different resonant states causing the dip-pair. We also depict the modulus squared of the wave function for the peak of the conductance curve at $L/M_n = 2.5$ in Figs. 5 (e) and (f). As can be seen from Figs. 5 (e) and (f), the maxima of electron locate on the opposite side of those of hole. The feature of these distributions shown in Figs. 5 (e) and (f) is very different from that shown in Figs. 5 (a)-(d). A very small increase of $L$ from $L/M_n = 2.5$ ($L = 100$) to $L/M_n = 2.58$ ($L = 103$), i.e., a slight change in the structure, brings about the large difference of the distribution of the quasiparticles because of the mixing of the different electronic states, which gives rise to the large change of the conductance [see Fig. 4(d)]. These characteristics are very different from those shown in the case of a finite bias voltage at $B = 0$, where the distributions of both quasiparticles in the central region for the conductance peak are similar to those for the neighboring dip of the conductance [see Fig. 2 (a)], which has been presented in Ref. 12.

Here, we are concerned with a $B$-dependence of the conductance briefly by considering the experimental situation, because of the difficulty of the change of $L$ in the experiment. We have calculated the conductance as a function of $B$ in the vicinity of the magnetic field where the dip occurs at $B = 4 \times 10^{-5}$ and $L/M_n = 2.58$. 
Fig. 5. The contour plot of modulus squared of the wave function of the quasiparticles around the central region in the case when the dip or peak occurs in the conductance in Fig. 4 (d). In (a), (c), and (e), $|u_{j,m}|^2$ is plotted. In (b), (d), and (f), $|v_{j,m}|^2$ is plotted. The values $L/M_n$ are set to be $L/M_n = 2.58$ for (a) (b), $L/M_n = 2.7$ for (c) (d), and $L/M_n = 2.5$ for (e) (f). The interval of the contour lines is 0.1.
Fig. 5. (continued)
[see Fig. 4 (d)], and the calculated results are given in Fig. 6. As shown in Fig. 6, the conductance minimum occurs at \( B = 3.9 \times 10^{-5} \). Here, we compare the magnitude of magnetic field in this calculation to the experimental value, although units of magnetic field are set to \( \phi_0/a^2 \) in this study. If we assume \( \lambda_F \approx 20 \text{nm} \), which corresponds to \( N_S = 2.3 \times 10^{12} \text{cm}^{-2} \) with \( N_S \) being the carrier density in the experiment reported in Ref. 2, then the value \( B = 4 \times 10^{-5} \) will correspond to \( B = 0.9 \text{T} \). In addition, the width of the wide region \( Ma = 90a \) corresponds to about 40nm. These values can be easily realized, and hence, it can be possible to observe the dip-pair in the conductance experimentally by tuning a magnetic field.

\[
\begin{align*}
\text{Conductance (Units of } 4e^2/h) \\
\text{vs. } B(\phi_0/a^2) \\
[\times 10^{-5}] \\
3 & \quad 4 \\
0.4 & \quad 0.2 & \quad 0.1 & \quad 0 & \quad 0.2 \\
3 \times 10^{-5} & \quad 2 \times 10^{-5} & \quad 1 \times 10^{-5} & \quad 0 \times 10^{-5} \\
\end{align*}
\]

Fig. 6. The \( B \)–dependence of the conductance in the vicinity of \( B = 4 \times 10^{-5} \) for \( L/M_n = 2.58 \) at \( eV = 0 \).

### 3.3. Influence of disorder

Next, we consider the effect of disorder in the vicinity of the interface on the transport phenomena in a magnetic field, since even small amounts of impurity are known to affect transport properties. We introduce a square shaped disorder region \( M \times D \) in the normal-conductor as shown in inset of Fig. 7. The on-site potential \( v_{j,m}^p \) is given randomly in the range of \( -V_{\text{dis}}/2 < v_{j,m}^p < V_{\text{dis}}/2 \) in addition to that given in eq. (36). The parameters \( \mu_S \) and \( \Delta \) are \( \mu_S = 6\mu_N \) and \( \Delta = 0.1\mu_S \), which are same values used previously. Figure 7 shows the calculated conductances for several values of \( V_{\text{dis}} \). As can be seen from Fig. 7, regardless of the introduction of the disordered region, the change of the shape of the conductance curves is small. This is because the classical cyclotron radius \( l_D (= 1064) \) and \( \lambda_F \) are very large. Thus, the effect of such a short range disorder on the conductance is small in the parameter region used in this calculation, although the amplitude of the conductance reduces slightly due to the effect of disorder.

In reality, little influence of the disorder indicates that the resonant transmis-
Conductance(Units of $4e^2/h$)

Fig. 7. The conductances through the disordered region are plotted as a function of $L/M_n$. The solid, dotted, and broken lines show the conductances for $V_{dis}=0$, $V_{dis}/\mu_N=0.5$, and $V_{dis}/\mu_N=1$, respectively. (a) $B=0$ and (b) $B=4 \times 10^{-5}$. Inset draws the schematic figure of the system in this case (see text). The closed region shows the superconducting region. The shaded region shows the disordered region. The length $D$ equals to $80a (=2M_n)$.

Conduction structure in such a complex sample structure may be detectable. This is also suggested from the fact that the resonant structure, e.g., the resonance of the step edge in a superconducting quantum point contact, where the width of the constriction is less than 100nm,\(^2\) has already been observed by the recent advance of nanotechnology.

3.4. Conductance of a sample without the narrow region

To check that the occurrence of the dip-pair is due to the NWS geometry in a magnetic field, we have calculated the conductance for a sample with a simple structure. In this case, the sample does not contain the narrow region, but a potential barrier at $j=1$. The potential profile is

$$v_{j,m}^p = \begin{cases} 12\mu_N & \text{at sites } j = 1, 1 \leq m \leq M \\ 0 & \text{otherwise} \end{cases}$$

In this case, the variable $L$ equals $L = N - 1$. The parameters $\mu_S$ and $\Delta$ are same as those used previously, i.e., $\mu_S = 6\mu_N$ and $\Delta = 0.1\mu_N$. Figure 8 shows the calculated conductance for $B = 0$ and $B = 4 \times 10^{-5}$ which have same magnitudes as those used in the case of the NWS geometry in Sec. 3.2. The magnitude of $\mu_j$ is identical with those used in Sec. 3.2. In Fig. 8, we can see several peaks which can be attributable to resonant tunneling of the Andreev and normal reflections. We assign the peaks to the mode-numbers by estimating the wavelength, and the mode-number of each peak is indicated in Fig. 8. As shown in Fig. 8, the peak structure for $B = 4 \times 10^{-5}$ is almost the same as that for $B = 0$ except that the locations of peaks around $L = 105$ and $L = 110$ for $B = 4 \times 10^{-5}$ are slightly
different from those for $B = 0$. However, as we have expected, dips are never seen in the conductance curve at $B = 4 \times 10^{-5}$. The magnetic field $B = 4 \times 10^{-5}$ is found to give little influence on the conductance peak in this geometry, although this value of magnetic field causes large change of the conductance curve in the case of NWS geometry as shown in Sec. 3.2. The abrupt change of the width of the wire at $j = N_{\text{narrow}} + 1$, which is treated in Sec. 3.2, can be essential for the appearance of the dip-pair in a magnetic field.

![Fig. 8](image_url)

**Fig. 8.** The conductances are plotted as a function of $L$ for $B = 0$ (dotted line) and $B = 4 \times 10^{-5}$ (solid line) for the sample not containing the narrow region at $eV = 0$. The numbers in the graph indicate the numbers of the subband causing the peaks in the conductance. Inset shows the schematic figure of the system in this case (see text).

### 3.5. Conductance of a normal NW wire

We also investigate the conductance of a sample with NW geometry which does not contain the superconductor, but another normal material, to confirm the role of the N-S interface for the occurrence of the dip-pair in a magnetic field. The sample geometry considered is almost the same as that treated in the case of NWS geometry. In this case, instead of the NS interface, we use the abrupt change of the chemical potential in the wide region. We postulate the presence of the insulator at the interface where the chemical potential changes abruptly. However, in practice, we do not put the insulator at the interface for simplicity, to elucidate the effect of the geometrical structure clearly. In this case, Eq. (12) is replaced by

$$
(P_{j})_{l,m} = \begin{cases} 
  p_m & (l = m = 1, \ldots, M) \\
  -p_m^{*} & (l = m = M + 1, \ldots, 2M) \\
  0 & \text{otherwise}
\end{cases}
$$

(38)
Resonant reflections through a normal-conductor–superconductor interface

$\Delta$ equals zero in whole region in this case. The values of $\mu_j$ and $v^p_{j,m}$ are same as those used in Sec. 3.2, which are given by Eqs. (3) and (36). Figure 9 shows the calculated conductance for $B = 0$ and $B = 4 \times 10^{-5}$ by the Landauer formula and the transfer matrix method. As shown in Fig. 9, the structure of the conductance curve at $B = 0$ is almost the same as that of the NWS geometry [see Fig. 2 (b)], although the magnitude of the conductance is almost half of those in the NWS case. When $B = 4 \times 10^{-5}$, the conductance curve changes considerably from that at $B = 0$ in the region $L/M_n \geq 2.2$ with $L = N - N_{\text{narrow}}$. In this case, the transmission of the first mode in the narrow region to the second mode in the wide region is permitted by the applied magnetic field, and hence, the resonances of the second mode in the wide region can take place in addition to the resonances of the third mode, which brings additional peaks in the conductance curve. As a consequence, the conductance curve at $B = 4 \times 10^{-5}$ shows peaky structures (see Fig. 9). However, there is no dip in the conductance curve in Fig. 9.

![Fig. 9](image)

Fig. 9. The conductances are plotted as a function of $L/M_n$ for $B = 0$ (dotted line) and $B = 4 \times 10^{-5}$ (solid line) for the normal NW wire not containing the superconducting region (see text). The chemical potential changes abruptly at $j = N$, i.e., $\mu_j = -\mu_N + 4t$ for $j \leq N$ and $\mu_j = -6\mu_N + 4t$ for $j > N$.

4. Summary

In summary, we have investigated the transport phenomena through a junction of the NS interface in the regime $eV < \Delta$ using the transfer matrix method with the stabilized iterative technique. We have examined the electronic state of quasiparticles when the resonance occurs. For the case of the NWS geometry, in a finite bias, each peak changes to dip-pair when the critical bias voltage for the corresponding resonant level is applied. The local charge density in the central region is much enhanced by the resonance of the electron-like or hole-like quasiparticle. As for the effect of a magnetic field on the conductance, we have also found that the peak in the conductance curve changes to the dip-pair. The wave function at the resonant
state indicates the mixing of the first and the second modes: the splitting of the dip into two is considered to be due to the mixing of two states with different parities, which gives rise to two different resonant states. The feature of the resonant state in a magnetic field at $eV = 0$ is very different from that in a finite bias voltage at $B = 0$. Moreover, the characteristics of the conductance curve of the NWS system in a magnetic field is found to be very different from those of the normal NW system with the similar geometry. The resonant state is very sensitive to the slight change of the structure, which leads to the change of the resonant reflection to the resonant transmission under a finite magnetic field.

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References