Spatial Distribution of Nonequilibrium Current in a Magnetic Field

Shinji Nonoyama and Akira Oguri

Faculty of Education, Yamagata University, Yamaqata 990-8560
Department of Material Science, Faculty of Science, Osaka City University, Osaka 558-8585

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The spatial distribution of the nonequilibrium current and charge distributions in a series of two point contacts are studied in the presence of a magnetic field making use of a recursive Green function method based on the Keldysh formalism. Numerical results show that the structure of vortices appearing in between two barriers depends strongly on the applied bias voltage. The chemical potential dependence of the total current is also investigated.

KEYWORDS: nonlinear transport, Keldysh’s Green function, local current, recursive method

§1. Introduction

The development of the semiconductor microprocessing technology has made it possible to fabricate low-dimensional quantum systems with various structures. In these mesoscopic systems, nonlinear transport phenomena have become a subject of current interest. For instance, the Aharonov-Bohm effect on the conductivity of the semiconductor mesoscopic system comprising two tunnel junctions has observed under finite bias voltages. Also, the bias dependence on the spin-polarized transport for the ferro-metal-semiconductor composite system has been discussed. However, despite the significance of the experiments of the nonlinear response in a magnetic field, theoretical approaches seem to be still in developing stages because of a difficulty in treating nonequilibrium quantum systems. Thus, it seems important for us to examine various numerical approaches to driven systems.

In the previous report, making use of a recursive Green function method based on the Keldysh formalism, we have studied the spatial distribution of the nonequilibrium current and the temperature dependence of the nonlinear current-voltage (I-V) characteristics in a series of two point contacts in the absence of a magnetic field. The purpose of this work is to examine the nonequilibrium transport in the presence of a magnetic field. To this end, we consider a system modeled on a two-dimensional tight-binding lattice, and compute the spatial distribution of current and charge by extending the recursive method for the Keldysh Green function to treat the effects of the magnetic field. We also study the chemical potential dependence of the total current. In the present study, we concentrate on a noninteracting case for simplicity. The effects of interaction within a mean field level can be included in a straightforward way by extending a treatment examined in a linear response case.

In §2, the details of model and method used in our calculation will be presented. In §3, we will show the spatial distribution of the current, and discuss the magnetic field dependence of the nonequilibrium current. A summary will be given in §4.

§2. Model and Method

We consider a quantum wire described by a two-dimensional tight-binding model as illustrated in Fig. 1, in which the lattice sites are labeled with the coordinate (i, j). The wire is assumed to be infinitely long in the x-direction, but in the y-direction it is assumed to be finite and consists of M lattice sites. Along the x-direction, the system consists of three parts; a finite central region at 1 ≤ i ≤ N, and two semi-infinite leads at −∞ < i ≤ 0 and N + 1 ≤ i < +∞. The Hamiltonian is given by

\[ H = -t \sum_{i=-\infty}^{\infty} \sum_{j=1}^{M-1} \left( C_{i,j+1} C_{i,j} + H.c. \right) \]

\[ -t \sum_{i=-\infty}^{\infty} \sum_{j=1}^{M-1} \left( C_{i+1,j}^\dagger C_{i,j} P_j^* + H.c. \right) \]

\[ + \sum_{i=1}^{N} \sum_{j=1}^{M} v_{i,j} C_{i,j}^\dagger C_{i,j} + \sum_{i=-\infty}^{\infty} \sum_{j=1}^{M} \phi_i C_{i,j}^\dagger C_{i,j}, \]

where \( C_{i,j}^\dagger \) is the creation operator for an electron at the lattice site (i, j), and \( P_j \) is the hopping matrix element for the nearest-neighbor sites. In above, a free boundary condition is assumed in the y-direction, and thus the hopping matrix elements connecting the sites at \( j = 1 \) and \( j = M \) are taken to be zero. When a uniform magnetic field \( B \) is applied along the z-direction, the Peierls phase factor can be chosen as \( P_j = \exp[2\pi i B (j - (M + 1)/2)] \), where \( \tilde{B} = Ba^2/\phi_0 \) and \( \phi_0 = (h/e) \) is the magnetic flux quantum. We will use the lattice constant \( a \) as the unit of the length in what follows. In the last two terms of eq. (2.1), \( v_{i,j} \) is the potential barrier for the point contacts, and \( \phi_i \) is the electrostatic potential due to an applied electric field. For a uniform field applied in the x-direction, we assume \( \phi_i \) to be

\[ \phi_i = \begin{cases} eV, & i \leq x_L \\ eV \frac{x_R - i + 1}{x_R - x_L + 1}, & x_L + 1 \leq i \leq x_R \\ 0, & x_R + 1 \leq i \end{cases} \]
where $V$ is the bias voltage. We will choose $x_L$ and $x_R$ to be $0 \leq x_L < x_R \leq N$ in order to examine the current distribution outside the two point contacts located at $x_L$ and $x_R$ in §3.

For studying the transport properties in a nonequilibrium state, we now introduce $M \times M$ matrix Green functions, $G^+(i,i')$, $G^-(i,i')$, and $F(i,i')$, the matrix elements of which are given, respectively, by

$$G^+_{i,i'}(i,i') \equiv -i \int_{-\infty}^{\infty} dt \langle \hat{C}_i(t), \hat{C}^+_j(0) \hat{D}_j(t) \rangle e^{i\omega t},$$

$$G^-_{i,i'}(i,i') \equiv i \int_{-\infty}^{\infty} dt \langle \hat{C}_i(t), \hat{C}^-_j(0) \hat{D}_j(t) \rangle e^{i\omega t},$$

$$F_{i,i'}(i,i') \equiv -i \int_{-\infty}^{\infty} dt \langle \hat{C}_i(t), \hat{C}^+_j(0) \hat{C}^-_j(t) \rangle e^{i\omega t}. \quad \text{(2.3)}$$

Here $C_i(t) \equiv \hat{e}^{iHt}C_i \hat{e}^{-iHt}$, $\hat{\theta}(t)$ is the step function, the bracket (curly bracket) denotes the commutator (anticommutator), and the index for the $\omega$ dependence is omitted in the left-hand side in order to simplify the notation. In the Keldysh formalism, the average $\langle \cdot \cdot \cdot \rangle$ is taken by using the density matrix at $t = 0$. Initially, at $t = -\infty$, two leads and the sample at the center are disconnected, and the left and right leads are in its own thermal equilibrium with the chemical potentials $\mu_L$ and $\mu_R$, respectively, with $\mu_{LR} \equiv \mu_R + eV$. Then, by the adiabatic switching-on of the hopping matrix element connecting the leads and the sample, the time evolution of the density matrix is determined. The perturbation expansion for the Green function can be described by using a matrix formulation. Specifically, in our case, it is convenient to introduce $2M \times 2M$ matrices $\tilde{G}(i,i')$ and $\tilde{\mathcal{T}}$;

$$\tilde{G}(i,i') = \begin{bmatrix} 0 & G^-(i,i') \\ G^+(i,i') & F(i,i') \end{bmatrix}, \quad \tilde{\mathcal{T}} = \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix}. \quad \text{(2.6)}$$

Here $P$ is an $M \times M$ diagonal matrix whose element is given by the Peierls phase factor $P_j$. In this representation the Dyson equation can be written as

$$\tilde{G}(i,i') = \tilde{G}_0(i) \delta_{i,i'} - \tilde{G}_0(i) \tilde{\mathcal{T}} \tilde{G}(i+1,i') - \tilde{G}_0(i) \tilde{\mathcal{T}}^\dagger \tilde{G}(i-1,i'). \quad \text{(2.7)}$$

Here $1 \leq i, i' \leq N$, and $\tilde{G}_0(i)$ is the Green function for the isolated column at $x = i$.

Using eq. (2.7), the intra-column Green function $\tilde{G}(i,i')$ can be obtained from a set of recursion formulas analogous to those for equilibrium systems

$$\tilde{G}(i,i) = \left[ \tilde{G}_0(i) - \tilde{\mathcal{T}}^\dagger \tilde{G}(i-1) \tilde{\mathcal{T}} - \tilde{\mathcal{T}} \tilde{G}(i+1) \tilde{\mathcal{T}}^\dagger \right]^{-1}, \quad \text{(2.8)}$$

$$\tilde{G}_L(i) = \left[ \tilde{G}_0(i) - \tilde{\mathcal{T}}^\dagger \tilde{G}_L(i-1) \tilde{\mathcal{T}} - \tilde{\mathcal{T}} \tilde{G}_R(i+1) \tilde{\mathcal{T}}^\dagger \right]^{-1}, \quad \text{(2.9)}$$

$$\tilde{G}_R(i) = \left[ \tilde{G}_0(i) - \tilde{\mathcal{T}}^\dagger \tilde{G}_R(i+1) \tilde{\mathcal{T}} - \tilde{\mathcal{T}} \tilde{G}_L(i-1) \tilde{\mathcal{T}}^\dagger \right]^{-1}, \quad \text{(2.10)}$$

where $1 \leq i \leq N$. The initial conditions for the recursive relations eqs. (2.9) and (2.10) are given at boundary $i = 1$ and $N$, respectively. Specifically, the matrices $\tilde{G}_L(0)$ and $\tilde{G}_R(N+1)$ are given by the Green functions for the unconnected leads on the left and right as

$$\tilde{G}_L(0) = \begin{bmatrix} 0 & G^+_L(0) \\ G^-_L(0) & F_L(0) \end{bmatrix}, \quad \text{(2.11)}$$

$$\tilde{G}_R(N+1) = \begin{bmatrix} 0 & G^+_R(N+1) \\ G^-_R(N+1) & F_R(N+1) \end{bmatrix}. \quad \text{(2.12)}$$

Furthermore, the diagonal parts $F_L(0)$ and $F_R(N+1)$ are written in terms of the retarded and advanced functions

$$F_L(0) = (1 - 2f_L) \left[ G^+_L(0) - G^-_L(0) \right], \quad \text{eq. (2.13)}$$

$$F_R(N+1) = (1 - 2f_R) \left[ G^+_R(N+1) - G^-_R(N+1) \right], \quad \text{eq. (2.14)}$$

where $f_L, f_R \equiv \left[ \text{e}^{\beta(\omega - \mu_{LR})} + 1 \right]^{-1}$ with $\beta$ being the inverse temperature $1/T$. Thus, the information of the density matrix is brought into the formulation through $F_L(0)$ and $F_R(N+1)$. The explicit expressions of $G^+_L(0)$ and $G^+_R(N+1)$ can be obtained by using the method described in ref. 14. In the sample region, the diagonal part of $\tilde{G}_0(i)$ can be treated as zero;  

$$\tilde{G}_0(i) = \begin{bmatrix} 0 & G_0(i) \\ G_0(i) & 0 \end{bmatrix}, \quad \text{eq. (2.15)}$$

where $G_0(i) \equiv [\omega 1 - H_0(i)]^{-1}$ with $H_0(i)$ being the Hamiltonian for the isolated $i$-th column. This is due to the fact that the physical quantities does not depend on the initial condition for the chemical potential in the sample region. The inter-column Green functions can also be obtained by using $2M \times 2M$ matrix recursion formulas;

$$\tilde{G}(i,i + 1) = -\tilde{G}(i,i) \tilde{T} \tilde{G}_R(i+1), \quad \text{eq. (2.16)}$$

$$\tilde{G}(i,i - 1) = -\tilde{G}(i,i) \tilde{T}^\dagger \tilde{G}_L(i-1). \quad \text{eq. (2.17)}$$

So far, we have used $2M \times 2M$ matrix formulation because it enables us to write a set of equations in a compact form. However, for numerical computations, it is more efficient to use the $M \times M$ matrix recursion formulas obtained by performing the inversions of the partitioned matrices explicitly. Equations (2.8)-(2.10) are rewritten, in terms of $M \times M$ matrices, as

![Schematic illustration of the system. The potential barriers only exist in the central region (hatched region). The quantities $N$ and $M$ are the numbers of the lattice sites for the $x$ and $y$ directions of the central region.](image)
\[ G_L^{\pm}(i) = [G_0(i)^{-1} - \ell^2 P^* G_L^{\pm}(i-1)P]^{-1}, \]  
\[ G_R^{\pm}(i) = [G_0(i)^{-1} - \ell^2 P G_R^{\pm}(i+1)P]^{-1}, \]  
\[ F_L(i) = \ell^2 G_L^{\pm}(i) P^* F_L(i-1)P G_L^{\pm}(i), \]  
\[ F_R(i) = \ell^2 G_R^{\pm}(i) P F_R(i+1)P^* G_R^{\pm}(i), \]  
\[ G^{\pm}(i,i) = [G_0(i)^{-1} - \ell^2 P G_R^{\pm}(i-1)P]^{-1}. \]

Similarly, eqs. (2.16) and (2.17) are equivalent to \( M \times M \) matrix formulas;
\[ G^{\pm}(i, i+1) = -t G^{\pm}(i, i) P G_R^{\pm}(i+1), \]  
\[ G^{\pm}(i, i-1) = -t G^{\pm}(i, i) P^* G_R^{\pm}(i-1), \]  
\[ F(i, i+1) = -t [G^{\pm}(i, i) P F_R(i+1) + F(i, i) P G_R(i+1)], \]  
\[ F(i, i-1) = -t [G^{\pm}(i, i) P^* F_L(i-1) + F(i, i) P^* G_L(i-1)]. \]

It is convenient for studying current and charge distributions to introduce an \( M \times M \) matrix lesser function, \( G^{-}(i,i') \), the matrix element of which is defined by
\[ G^{-}(i,i') \equiv i \int_{-\infty}^{\infty} dt \langle C_{j,j'}(t) C_{i,i}(0) \rangle e^{i\omega t}. \]  
\[ \text{The matrix } G^{-}(i,i') \text{ is obtained from } G^{\pm}(i,i') \text{ and } F(i,i') \text{ using an identity} \]
\[ G^{-}(i,i') = \frac{1}{2} [F(i,i') - G^{\pm}(i,i') + G^{-}(i,i')] \]  
In terms of the lesser function, the local charge at site \((i,j)\) is expressed as
\[ \rho_{i,j} = \int \frac{d\omega}{2\pi} G^{\pm}_{i,j}(i,i). \]  
\[ \text{Similarly, the local current around } (i,j) \text{ is expressed as}^{11} \]
\[ I_{i,j}^L = \frac{e}{\hbar} \int \frac{d\omega}{2\pi} \left[ G^{\pm}_{j,j}(i,i+1)P^*_{j} - G^{-}_{j,j}(i+1,i)P_{j} \right], \]  
\[ I_{i,j}^R = \frac{e}{\hbar} \int \frac{d\omega}{2\pi} \left[ G^{\pm}_{j+1,j}(i,i) - G^{-}_{j+1,j}(i,i) \right]. \]

In the presence of the magnetic field, it is suitable for numerical integration with respect to \( \omega \) to use eqs. (2.31)–(2.32) rather than the alternative expressions in terms of \( F^S \). This is because the lesser function has an upper cut-off \( \mu_L \) in \( \omega \) dependence through the distribution function \( f_L \) (we assume \( \mu_L \geq \mu_R \)). Also, when the time-reversal symmetry is broken, the equivalence of the alternative expressions for the local current are justified after the integration is performed in the whole region of \( \omega \). Specifically, the total current in the \( x \) direction, \( I_{tot} \equiv \sum_{j=1}^{M} I_{i,j}^L + I_{i,j}^R \), can be expressed simply in terms of \( G^{\pm,1,2,11,12} \).

\[ I_{tot} = \frac{e}{\hbar} \int d\omega (f_L - f_R) \text{Tr} \left[ G_L^{+}(1,N) \Gamma_R G^{-}(N,1) \right]. \]

Here \( \text{Tr} \) denotes the trace for \( M \times M \) matrices, \( \Gamma_L \equiv i\hbar^2 P^* [G_L(0) - G_L(0)]P \) and \( \Gamma_R \equiv i\hbar P [G_R(N+1) - G_R(N+1)]P^* \). At \( T = 0 \), the integration region of eq. (2.33) is restricted to be \( \mu_L \leq \omega \leq \mu_L \) due to the factor \( f_L - f_R \). In the next section, we apply the method described above to a nonequilibrium current through a series of two point contacts.

### §3. Numerical Results and Discussion

In this section, we will present the numerical results of the \( I - V \) characteristics and the spatial distribution of the currents and discuss the effect of the bias voltages and magnetic fields on the transport properties. In what follows, we take the transfer integral \( t \) as a unit of the energy. For the computation of the nonequilibrium current, the integral in eqs. (2.30)–(2.33) is replaced by Simpson’s sum, and the mesh is taken to be typically \( \Delta \omega = 10^{-5} \). The chemical potential \( \mu_R \) is measured from \(-4t\), i.e., the energy corresponding to the bottom of the conduction band for an infinitely large square lattice is taken to be the origin. For simplicity, we consider only the case \( T = 0 \) in the following calculations.

#### 3.1 Current distribution without potential barrier

Prior to the investigation of the current distribution for a sample geometry with a potential barrier, we examine the current distribution of a quantum wire without potential barrier in a magnetic field. We consider a system in a linear response regime with \( N = 28, M = 20, \mu_R = 0.5, \) and \( eV = 0.001 \). We apply the electric field to the region \( 5 \leq i \leq 24 \) setting \( x_L = 5 \) and \( x_R = 23 \) for the latter convenience. Note that two leads and the central region are connected at \( i = 0 \) and \( i = 29 \) in this case. In this subsection, we consider a high magnetic field case at \( B = 0.05 \). To see a relative location of \( \mu_R (=0.5) \) in energy bands, we have plotted the wave-number dependence of the eigenvalues for \( eV = 0 \) in Fig. 2. In this high magnetic field, the Landau levels are formed, and consequently, \( \mu_R \) is situated in between the first and the second Landau levels (see Fig. 2).

In Fig. 3(a) we show the calculated current distribution. In this case, the currents are the sum of the edge and bulk currents. The edge current flows from left (right) to right (left) along the lower (upper) edge. However, the bulk current rotates clockwise, and hence, that flows totally from right (left) to left (right) near the lower (upper) edge. By the contribution of a large amount of the bulk current, the edge-state currents are canceled out so that the direction of the current near the edge in Fig. 3(a) is seen to be opposite to the current of the edge state. We also show the current of the edge state in Fig. 3(b), where the range of integration with respect to \( \omega \) defined by eqs. (2.31) and (2.32) is restricted to be in between \( \mu_L \) and \( \mu_R \), although the magnitude of the arrows in Fig. 3(b) is enlarged 20-fold over that in Fig. 3(a).
1148 Shinji Nonoyama and Akira Oguri (Vol. 69, current and the structure of the current distribution in a magnetic field for such a system. In this case, we also consider a system with $N = 28$ and $M = 20$, and choose the on-site potential to be finite, $v_{i,j} = 1.0$, for the sites which constitute the two point contacts, i.e., all the sites on the column $i = 6, 7, 22, 23$ except those on the row $j = 9, 10, 11, 12$ [see the inset of Fig. 4 and Fig. 6]. The electric field is applied to the region in between the two point contacts, i.e., the region $5 \leq i \leq 24$ ($x_L = 5$ and $x_R = 23$).

First, we present numerical results for the total current. In Fig. 4 the total current as a function of $\mu_R$ for (a) $\vec{B} = 0$, (b) $\vec{B} = 0.001$, (c) $\vec{B} = 0.005$, (d) $\vec{B} = 0.01$, and (e) $\vec{B} = 0.05$ in the linear response regime. The applied bias voltage is $eV = 0.001$. (a) The inset shows a schematic figure of the two point contacts. For details of the model see the text.

### 3.2 Two point contacts

Next, we consider a current through a series of two point contacts, which is modeled by arranging the values of $v_{i,j}$. In the previous report, we presented the numerical results for this system with two serial point contacts without magnetic fields. We showed the character such as the appearance of small vortices even without magnetic fields in the nonlinear response regime. In this study, we further explore the investigation of the total

![Fig. 2. The wave number $k_x$ dependence of the eigenvalues for the case of $v_{i,j} = 0$ and $\Phi_i = 0$ at $\vec{B} = 0.05$. The dotted line denotes the chemical potential $\mu_R$.](image)

![Fig. 4. The total current as a function of $\mu_R$ for (a) $\vec{B} = 0$, (b) $\vec{B} = 0.001$, (c) $\vec{B} = 0.005$, (d) $\vec{B} = 0.01$, and (e) $\vec{B} = 0.05$ in the linear response regime. The applied bias voltage is $eV = 0.001$. (a) The inset shows a schematic figure of the two point contacts. For details of the model see the text.](image)

![Fig. 3. (a) The current distribution for $eV = 0.001$ of the model without potential barrier. (b) The current distribution of the edge states, where the range of integration is restricted in between $\mu_L$ and $\mu_R$. The magnitude of the arrows is enlarged 20-fold over that in (a) (see text).](image)
presence of many resonant states in the energy spectrum between $\mu_R$ and $\mu_L(= \mu_R + eV)$ makes the current be less sensitive to the change of $\mu_R$. We have plotted the total current for several values of $\tilde{B}$ at $eV = 0.5$ in Fig. 5. The peak structure of the resonant tunneling is smeared out by the high bias voltage so that little change of the curve structure can be seen as a magnetic field increases.

Next we investigate the current distribution. Figs. 6(a), 6(b) and 6(c) show the results in a linear response regime ($eV = 0.001$) for $\tilde{B} = 0$, $\tilde{B} = 0.001$, and $\tilde{B} = 0.005$, respectively. In this case, the chemical potentials are set to be $\mu_R = 0.055$ for $\tilde{B} = 0$, $0.001$, and $\mu_R = 0.06$ for $\tilde{B} = 0.005$, where the resonances occur. The corresponding wavelength at $\tilde{B} = 0$ is estimated to be about 35 in units of the lattice constant $a$. In the case of $\tilde{B} = 0$ [see Fig. 6(a)], the current flowing through the point contact on the left spreads gradually till the center of the sample and then focuses at the point contact on the right. This feature can be determined by a standing wave due to the resonance. On the other hand, in the case of $\tilde{B} = 0.001, 0.005$ [see Figs. 6(b) and 6(c)], we can see a large vortex current in between the two point contacts. The appearance of this large vortex is the result of the contribution of the current, the energy of which is below $\mu_L$. Also, in Figs. 6(b) and 6(c), the center of the vortex is seen to shift to down from the center of the wire by the Lorentz force. The shift of the vortex accompanies the corresponding sift of the charge. In the stationary state, the Lorentz force is balanced by the reaction from the hard-wall boundary, and we will give the description of the charge distribution later [see Fig. 7(a)]. In the finite magnetic field, the time-reversal symmetry is broken so that we must perform the integration with respect to $\omega$ defined by eqs. (2.31) and (2.32) from the bottom of the energy band to $\mu_L$ at $T = 0$. This is quite different from the calculation of the total current. If we calculate the current distribution only contributing to the total current by the integration the range of which is restricted for $\mu_R \leq \omega \leq \mu_L$ at $\tilde{B} = 0.001$, we obtain a distribution as shown in Fig. 6(d). This distribution is somewhat resemble to that shown in Fig. 6(a) at $\tilde{B} = 0$.

We have also examined the case $eV = 0.5$ where the $I-V$ characteristic is nonlinear, and the results are shown in Figs. 6(e), 6(f) and 6(g) for $\tilde{B} = 0, \tilde{B} = 0.001$, and $\tilde{B} = 0.005$, respectively. The current distributions become to be complicated, compared with those shown in Figs. 6(b) and 6(c). In particular, there are many small vortices in Fig. 6(g). Those current distributions are determined by the sum of the currents in the state for $\omega \leq \mu_L$. The superposition of the currents for many single-electron levels brings about such a complicated structure of the distribution, and thus, it is impossible to assign the origin of the appearance of the vortex to the contribution of a certain unique single-electron level.

Furthermore, we shall present numerical results of a charge distribution for above systems in Figs. 7(a)–7(d). When $eV$ is small ($eV = 0.001$), we can see the maximum of the charge density in between two point contacts, which is due to the resonance, as shown in Figs. 7(a) ($\tilde{B} = 0$ and $\mu_R = 0.055$) and 7(b) ($\tilde{B} = 0.005$ and $\mu_R = 0.06$). These two structures of the charge distributions in Figs. 7(a) and 7(b) are resemble, although the current distributions indicate quite different structures as described before [see Figs. 6(a) and 6(c)]. Also, in the finite magnetic field, the maximum of the charge shifts to down slightly from the center of the quantum wire. When $eV = 0.5$, the amount of the charge in the region between two point contacts becomes very small, compared with those in the lead wires [see Figs. 7(c) ($\tilde{B} = 0$ and $\mu_R = 0.055$) and 7(d) ($\tilde{B} = 0.005$ and $\mu_R = 0.06$)]. This reflects that the velocity of the electron in the central region is very large. Moreover, in Fig. 7(d), several local accumulations of the charge can be seen, and those are distributed asymmetrically. This is attributable to the presence of many resonant levels below $\mu_L$.

§4. Summary

In summary, we have studied the nonequilibrium current and charge distributions, and nonlinear $I-V$ characteristics, in a quantum wire containing a series of two point contacts in a magnetic field. We have found some interesting behaviors such as the existence of large vortices in the linear response regime and the asymmetric distribution of small vortices in the nonlinear regime. The appearance of these vortices is the result for the sum of the currents including those for the resonant state below $\mu_L$, and the charge distribution is not necessarily reflect the current distribution. For obtaining the numerical results, we have used the recursive Green function method on the basis of the Keldysh perturbation theory. Although the self-consistent treatment of the electrostatic potential has been ignored in the present work, it can be included as the method described in ref. 13. The calculation along this line will be discussed in future.

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Fig. 6. The current distributions for the linear and nonlinear response cases. The bias voltages are set to be $eV = 0.001$ for (a), (b), (c) and (d), and $eV = 0.5$ for (e), (f) and (g). The magnitudes of the magnetic field applied are $B = 0$ for (a) and (e), $B = 0.001$ for (b), (d) and (f), and $B = 0.005$ for (c) and (g). The open circles (○) denote the sites which constitute the potential barrier. The cross ($\times$) is a label for indicating the center of a vortex and just a guide for the eyes in (d). The magnitudes of the arrows are enlarged by factors (a) 3, (b) 0.5, (c) 0.08, (d) 2, (e) 0.4, (f) 0.4, and (g) 0.08 over that in Fig. 2 (b). (g) The range of the integration is restricted to be $\mu_R < \omega < \mu_L$. 
Fig. 7. The contour maps of the local charge density. The bias voltages are set to be $eV = 0.001$ for (a) and (b), and $eV = 0.5$ for (c) and (d). The magnitudes of the magnetic field applied are $B = 0$ for (a) and (c), and $B = 0.005$ for (b) and (d). The chemical potentials are set to be $\mu_R = 0.055$ for (a) and (c), $\mu_R = 0.06$ for (b) and (d). The interval of the contour lines is $5 \times 10^{-4}$. The lowest value of the contour lines is $2 \times 10^{-4}$.

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